

Closing **today** at 11pm: HW_1A,1B,1C

Closing next Wed: HW_2A,2B,2C

Very Quick review of foundations:

Definition of Definite Integral:

If $\Delta x = \frac{b-a}{n}$ and $x_i = a + i\Delta x$, then

$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x$$

= "signed" area between $f(x)$ and the x-axis from $x=a$ to $x=b$.

FTOC(1): Areas are antiderivatives!

$$\frac{d}{dx} \left(\int_a^x f(t) dt \right) = f(x)$$

FTOC(2): $F(x)$ any antiderivative of $f(x)$,

$$\int_a^b f(x) dx = F(b) - F(a)$$

Entry Task: Use the FTOC (Part 2), to evaluate each definite integral below:

$$\int_0^4 e^x + \sqrt{x^3} dx$$

$$\int_3^6 \frac{4}{x} - \frac{2}{x^2} dx$$

5.4 The Indefinite Integral and Net/Total Change

Def'n: An **indefinite integral** of $f(x)$ is defined to be the general antiderivative of $f(x)$. And we write

$$\int f(x)dx = F(x) + C,$$

where $F(x)$ is one antiderivative of $f(x)$.

Net Change and Total Change

Assume an object is moving along a straight line (up/down or left/right).

Let $s(t)$ = 'location of object at time t '

$v(t)$ = 'velocity of an object at time t '

positive $v(t)$ means moving up/right

negative $v(t)$ means moving down/left

The FTC (part 2) says

$$\int_a^b v(t) dt = s(b) - s(a)$$

i.e. 'integral of rate of change of dist.'

= 'net change in distance'

Displacement = net change in distance

$$= \int_a^b v(t) dt = s(b) - s(a)$$

This is just the difference between where the object was standing at $t = a$ and where it was standing at $t = b$.

Total Distance traveled by the object

$$= \int_a^b |v(t)| dt$$

To compute 'total distance' follow these steps:

1. Solve $v(t) = 0$.
(This tells you places where $v(t)$ might switch between positive and negative)
2. Break up the problem into separate integrals at the places from step 1.
3. Evaluate each integral (without the absolute values) and **add them up as positive numbers** in the end.